Two-photon decay of heavy hadron molecules

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(Dated: July 27, 2010)

We discuss the two-photon decay width of heavy hadron molecules and study the dependence on the constituent meson masses and on the binding energy. In addition finite size effects due to the extended structure of the bound state are shown to have a strong influence on the predictions for this decay width.

Conventional quark-antiquark constituent quark model descriptions of the meson spectrum are certainly incomplete and sometimes inadequate to match experimental observation. As a complementary or even alternative approach new structure assumptions have been invoked in the past. This concerns for example glueballs, quark-gluon hybrids, tetraquark configurations or even an admixture of some of these states with an ordinary quark-antiquark meson as suggested by direct applications of (as in lattice simulations) or in models motivated by the underlying theory of QCD [1, 2]. But also conventional configurations can populate the meson spectrum as quasi-nuclear nucleon-antinucleon bound states [3] or hadronic molecules, bound states of two or more mesons [4–6].

The idea that mesons can also form mesonic bound states traces back to the original work of [7, 8] with a first systematic study for the meson spectrum given in [9]. The concept of hadronic molecules is often invoked when observed meson masses are rather close to a respective two-meson threshold and when dynamical arguments can be found which suggest binding in these systems. Experimental candidates for these type of states are especially discussed in light of the experimental advances in the heavy flavor sector. A prominent example is the $X(3872)$ where a molecular interpretation is suggested [10].

A full interpretation of an observed meson resonance is not only based on a consistent description of its mass and $J^{PC}(I^G)$ quantum numbers, but also on a possible explanation of the observed decay and production modes of this respective state. Here the electromagnetic interaction plays a leading role since it is well understood even on a effective hadronic level and can be treated perturbatively. In particular, two-photon or, in general, radiative decays are considered diagnostic tools which are sensitive to the inner structure of the short-lived resonance [8].

The purpose of the present manuscript is the discussion of the two-photon decays of heavy hadron molecules. Our aim is to work out the influence of the binding energy and the constituent meson masses on quantities like couplings (of the molecular state to its constituent mesons) and electromagnetic decay amplitudes. Under certain circumstances, as for a local theory, the results are analytical and the dependence on the above mentioned quantities can be shown explicitly. Especially ratios of two-photon decay rates of bound states, also in different flavor sectors, provide a simple and clear estimate whether a molecular structure is likely or not.

For the physical or nonlocal case, which effectively models finite size effects of hadronic molecules, results can deviate sizably from the limiting case of a local theory. Here we find that, in general, the two-photon decay properties of heavy bound states are more sensitive to finite size effects than hadronic molecules of light mesons.

Several heavy meson and baryon states are considered good candidates for a molecular structure among them the $D_{s0}^*(2317)$, $D_{s1}(2460)$, $X(3872)$, $Y(3940)$, $Y(4140)$, $Y(4660)$ and $\Lambda_c(2940)$. For a review of the situation in the heavy meson sector we refer to [1, 4, 11].

For the present study of two-photon decays of heavy meson molecules we choose the framework of a phenomenological Lagrangian approach developed in [5, 12]. The bound state structure of hadronic molecules is set up by the compositeness condition [12, 13] which implies that the renormalization constant Z of the hadron wave function is set equal to zero or that the hadron exists as a bound state of its constituents. Decay processes of hadronic bound states are described by the coupling of the final state particles via meson-loop diagrams to the constituents of the molecular state [5, 6, 14]. First calculations for the two-photon decay widths of heavy hadron molecules as $Y(3940) = \{D^*D^{*\dagger}\}$ and $Y(4140) = \{D_s^*D_s^{*\dagger}\}$ have been performed in [14].

In this note we concentrate on hadronic bound states in the meson sector. We further restrict to S-wave hadron molecules with quantum numbers $J^{PC} = 0^{++}$ whose constituents are pseudoscalar or scalar charmed/bottom mesons. The coupling of the scalar molecular state H to its constituents is expressed by the phenomenological

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Lagrangian:

$$\mathcal{L}_H(x) = g_H H(x) J_H(x) \tag{1}$$

where g_H is the coupling constant, H is the field representing the hadronic molecule and $J_H(x) = \phi^\dagger(x)\phi(x)$ is the current carrying the quantum numbers of the molecule composed of the constituent mesons identified with the field ϕ. A nonlocal extension, which corresponds to the physical case of a hadronic molecule with a finite size, can easily be achieved by insertion of a vertex function $\Phi_H(y^2)$ into the current $J_H(x)$:

$$J_H(x) = \int d^4 y\, \Phi_H(y^2)\, \phi^\dagger(x+y/2)\, \phi(x-y/2) \tag{2}$$

where y is the relative Jacobi coordinate. Here $\Phi_H(y^2)$ is the correlation function describing the distribution of the constituents inside the molecular state H. In the present evaluations we use a Gaussian form with $\Phi_H(y^2)$: $\tilde{\Phi}_H(p_E^2/\Lambda_{H_i}^2) \doteq \exp(-p_E^2/\Lambda_{H_i}^2)$, where p_E is the Euclidean Jacobi momentum and Λ_{H_i} is a size parameter.

The coupling constant g_H is determined by the compositeness condition [12, 13], where $Z = 1 - g_H^2 \Sigma'(M^2) = 0$. Here, $\Sigma'(M^2) = d\Sigma(p^2)/dp^2|_{p^2=M^2}$ is the derivative of the mass operator $\Sigma(p^2)$ illustrated in Fig. 1, where M is the mass of the hadronic molecule. The generic expression for the mass operator of a bound state of two pseudoscalars is e.g. given in [6],

$$\Sigma(p^2) = \int \frac{d^4 k}{(2\pi)^4 i} \tilde{\Phi}(-k^2) S\left(k+\frac{p}{2}\right) S\left(k-\frac{p}{2}\right), \tag{3}$$

where $S(k) = (m^2 - k^2 - i\epsilon)^{-1}$ denotes the free propagator of the constituent meson.

In the following we first restrict to the case of a local coupling of the H state to its respective constituents. In case of a local interaction the vertex function is replaced by $\tilde{\Phi}(-k^2) = \exp(k^2/\Lambda_{H_i}^2) \equiv 1$. Then the coupling g_H can be expressed in the analytical form [6, 15]:

$$g_H^{-2} = \frac{1}{(8\pi m\zeta)^2} \left[\frac{\beta(\zeta)}{\sqrt{1-\zeta^2}} - 1 \right], \tag{4}$$

where $\zeta = M/(2m)$, $\beta(\zeta) = \arcsin(\zeta)/\zeta$ and m is the mass of the constituent meson.

Inclusion of the electromagnetic interaction in a gauge invariant way is discussed in our previous works [6, 16]. In the local approximation we deal with the two diagrams of Fig. 2. The extension to the nonlocal case leads to further diagrams, which need to be included in order to guarantee gauge invariance (a full discussion can be found in [6, 16]).

The gauge invariant form of the matrix element for the radiative transition in case of real photons reads as:

$$M_{\mu\nu} = (g_{\mu\nu} q_1 q_2 - q_{2\mu} q_{1\nu})\, g_{H\gamma\gamma}, \tag{5}$$

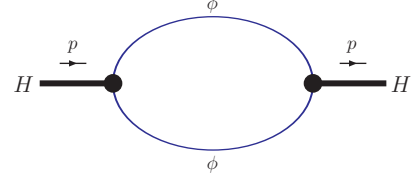where q_1 and q_2 are the 4-momenta of the photons. For the local case the expression for the effective coupling

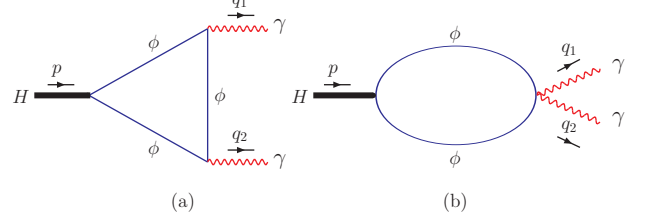FIG. 1: Mass operator of the hadronic molecule $H = (\phi\phi^\dagger)$.

FIG. 2: Diagrams contributing to the $H \to \gamma\gamma$ decay.

constant $g_{H\gamma\gamma}$ is given by the simple form:

$$g_{H\gamma\gamma} = \frac{g_H}{(4\pi m\zeta)^2} \left[\beta^2(\zeta) - 1 \right]. \tag{6}$$

However, this expression becomes more complicated and can only be solved numerically when including finite size effects (see e.g. Appendix of [6]). The radiative decay width of the 0^{++} molecule is calculated according to the formula

$$\Gamma(H \to \gamma\gamma) = \frac{\pi}{4} \alpha^2 M^3 g_{H\gamma\gamma}^2. \tag{7}$$

In the local limit we can write the decay width in terms of the quantity ζ by using Eq. (6)

$$\Gamma(H \to \gamma\gamma) = \frac{\alpha^2}{2\pi} m\, I(\zeta), \tag{8}$$

with

$$I(\zeta) = \zeta \sqrt{1-\zeta^2} \frac{(\beta^2(\zeta)-1)^2}{\beta(\zeta) - \sqrt{1-\zeta^2}}. \tag{9}$$

Since in the case of hadronic bound states the binding energy ϵ is small in comparison to the masses of the constituent mesons m we can perform an expansion of g_H, $g_{H\gamma\gamma}$ and $\Gamma(H \to \gamma\gamma)$ in $x = \epsilon/(2m)$, where $\zeta = 1 - x$. In order to guarantee an accurate approximation of the observables we need to include the leading (LO) and next-to-leading order (NLO) terms in the expansion of g_H and $g_{H\gamma\gamma}$. Therefore, the expansion of $\Gamma(H \to \gamma\gamma)$ includes three terms up to the next-to-next-to-leading order (NNLO) contribution.

The corresponding x-expansions of the quantities of interest are given by:

$$g_H = m\, I_H(x), \qquad g_{H\gamma\gamma} = m^{-1} I_{H\gamma\gamma}(x),$$

$$\Gamma(H \to \gamma\gamma) = \left(\frac{\pi\alpha}{4} \right)^2 m\, J_{H\gamma\gamma}(x), \tag{10}$$

where

$$I_H(x) = 8\sqrt{2\pi}\,(2x)^{1/4}\left(1 + \frac{2}{\pi}\sqrt{2x}\right) + \mathcal{O}(x^{5/4})\,,$$

$$I_{H\gamma\gamma}(x) = \frac{\sqrt{2\pi}}{8}\,(2x)^{1/4}\left(1 - \frac{4}{\pi^2} - \frac{2}{\pi}\left(1 + \frac{4}{\pi^2}\right)\sqrt{2x}\right) + \mathcal{O}(x^{5/4})\,,$$

$$J_{H\gamma\gamma}(x) = \sqrt{2x}\left(\left(1 - \frac{4}{\pi^2}\right)^2 - \frac{4}{\pi}\left(1 - \frac{16}{\pi^4}\right)\sqrt{2x}\right. $$
$$\left. + 2x\left(\frac{7}{8} + \frac{9}{\pi^2} - \frac{50}{\pi^4} + \frac{256}{\pi^6}\right)\right) + \mathcal{O}(x^2)\,. \quad (11)$$

In the next step we consider bound states of the pseudoscalar D, D_s and B mesons which have the following structure

$$|H_D\rangle = \frac{1}{\sqrt{2}}\left(|D^+D^-\rangle + |D^0\overline{D^0}\rangle\right),$$
$$|H_{D_s}\rangle = |D_s^+D_s^-\rangle \quad (12)$$
$$|H_B\rangle = \frac{1}{\sqrt{2}}\left(|B^+B^-\rangle + |B^0\overline{B^0}\rangle\right).$$

Note, the existence of bound states of two heavy pseudoscalar mesons was proposed before in Ref. [17] where hadronic molecules are dynamically generated in a coupled channel formalism. Based on the identifications of Eq. (12) additional flavor factors have to be considered in Eq. (10) which leads to:

$$g_{H_i} = m_i\,I_H(x_i)\,, \qquad g_{H_i\gamma\gamma} = \frac{c_{H_i\gamma\gamma}}{m_i}\,I_{H\gamma\gamma}(x_i)\,,$$

$$\Gamma(H_i \to \gamma\gamma) = \left(\frac{\pi\alpha}{4}\right)^2 m_i\,c_{H_i\gamma\gamma}^2\,J_{H\gamma\gamma}(x_i)\,, \quad (13)$$

where $x_i = \epsilon/(2m_i)$, $c_{H_i\gamma\gamma} = 1/\sqrt{2}$ for $i = D, B$ and 1 for $i = D_s$. From last equation follows the ratio of the two-photon widths of two different molecular states which is characterized by the respective constituent masses and binding energies. At leading order we deal with the simple expression:

$$\frac{\Gamma(H_B \to \gamma\gamma)}{\Gamma(H_D \to \gamma\gamma)} \sim \left(\frac{m_B\epsilon_B}{m_D\epsilon_D}\right)^{1/2}\,. \quad (14)$$

Varying the binding energy ϵ from 10 to 100 MeV which (besides the exception of the X(3872)) are typical values in the heavy meson sector, the two-photon decay widths are evaluated as

$$\Gamma(H_D \to \gamma\gamma) = 0.25 - 1.19\text{ keV}\,,$$
$$\Gamma(H_{D_s} \to \gamma\gamma) = 1.31 - 2.58\text{ keV}\,, \quad (15)$$
$$\Gamma(H_B \to \gamma\gamma) = 1.19 - 2.78\text{ keV}\,.$$

Since $\Gamma(H_i \to \gamma\gamma) \propto \sqrt{m\epsilon}$ the smaller value of the binding energy also results in a smaller radiative decay width.

Up to now the results for the coupling and the two-photon decay widths were given in the local limit. For the
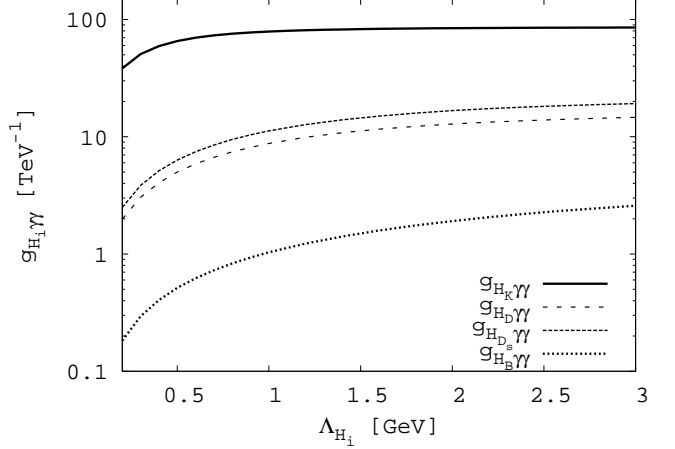

FIG. 3: Couplings $g_{H_K\gamma\gamma}$ and $g_{H_i\gamma\gamma}$ with $i = D, D_s, B$ in dependence on Λ_{H_i} for $\epsilon_K = 7.35$ MeV and $\epsilon_i = 10$ MeV.

physically appropriate description of an extended object, the hadronic molecule, finite size effects in terms of the vertex function have to be included. This will lead to a suppression of the couplings and the two-photon decay widths, hence the local case presents an upper limit for these quantities. In the following we study in addition to the influence of mass and binding energy the dependence on the size parameter Λ_{H_i}, which models the nonlocality.

The dependence of the couplings $g_{H_i\gamma\gamma}$ on the size parameter Λ_{H_i}, typically chosen in the range from 1 to 3 GeV, is illustrated in the logarithmic plots of Figs. 3-4 for different values of ϵ ranging from 10 to 100 MeV. For comparison we also include the coupling $g_{H_K\gamma\gamma}$ of $f_0(980)/a_0(980)$ to two photons with the well determined binding energy of $\epsilon = 7.35$ MeV. The local case, i.e. $\Lambda_{H_i} \to \infty$, characterizes the asymptotics of the curves. The convergence of the coupling $g_{H_i\gamma\gamma}$ towards the local or asymptotic value depends on the constituent meson masses. The coupling $g_{H_K\gamma\gamma}$ is almost stable with respect to variations of Λ_{H_i} near 1 GeV. The couplings of the heavy hadron molecules are more sensitive to finite size effects for values near $\Lambda_{H_i} \approx 1$ GeV, note that in Figs. 3-4 the dependence on Λ_{H_i} is displayed on a log scale. In fact, the coupling $g_{H_K\gamma\gamma}$ reaches 90% of the asymptotic value (local approximation) already at $\Lambda_{H_i} \approx 0.6$ GeV. In contrast, the couplings of heavy bound states approach 90% of the local value at around 4 GeV in case of the D and D_s-meson bound states and at about 9 GeV for the even heavier hidden-bottom molecule.

The behavior of the couplings $g_{H_i\gamma\gamma}$, which enter quadratically in the expression for the radiative decay width, shows that the suppression due to the size parameter Λ_{H_i} is larger in case of heavier constituents. Phenomenologically, the values for Λ_{H_i} tend to increase in case of heavy bound states. This behavior was already observed in earlier analyses in the framework of meson molecules [5, 6, 14] but also in case of baryons [18]. But still, for reasonable values of Λ_{H_i} heavier systems are
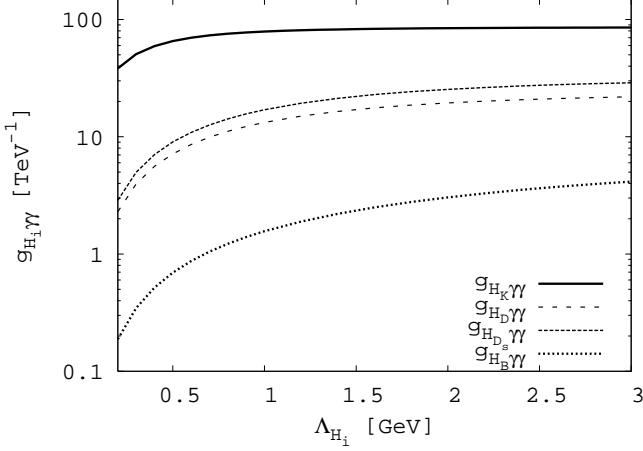
FIG. 4: Couplings $g_{H_K\gamma\gamma}$ and $g_{H_i\gamma\gamma}$ with $i = D, D_s, B$ in dependence on Λ_{H_i} for $\epsilon_K = 7.35$ MeV and $\epsilon_i = 100$ MeV.

strongly influenced by finite size effects as reflected in the two-photon coupling $g_{H_i\gamma\gamma}$.

In practice Λ_{H_i} should also depend in average on the binding energy. For instance, a small binding energy would lead to a loose bound state with a more extended structure than a strongly bound compact state. Therefore Λ_{H_i} should decrease with smaller binding energies, increasing the deviation from the local limit even more.

To quantify the inclusion of finite size effects for the two-photon decay widths we also give results for two values of Λ_{H_i}. For $\Lambda_{H_i} = 1$ GeV and $\epsilon = 10 - 100$ MeV we have

$$
\begin{aligned}
\Gamma(H_D \to \gamma\gamma) &= 0.05 - 0.34 \text{ keV}, \\
\Gamma(H_{D_s} \to \gamma\gamma) &= 0.32 - 0.68 \text{ keV}, \\
\Gamma(H_B \to \gamma\gamma) &= 0.05 - 0.12 \text{ keV}.
\end{aligned} \tag{16}
$$

When increasing Λ_{H_i} to 2 GeV the results are by a factor $2-3$ larger

$$
\begin{aligned}
\Gamma(H_D \to \gamma\gamma) &= 0.13 - 0.73 \text{ keV}, \\
\Gamma(H_{D_s} \to \gamma\gamma) &= 0.71 - 1.52 \text{ keV}, \\
\Gamma(H_B \to \gamma\gamma) &= 0.18 - 0.44 \text{ keV}.
\end{aligned} \tag{17}
$$

These results should be compared to the local limits of Eq. (15).

From previous discussions it should be clear that finite-size effects are quite important for a quantitative determination of the radiative decay width of heavy hadron molecules. The same observation holds for molecular states composed of vector mesons [14], where we showed that for $\Lambda_{H_{D^*}} = \Lambda_{H_{D_s^*}} = 1 - 2$ GeV the radiative widths of the molecules $Y(3940) = \{D^*D^{*\dagger}\}$ and $Y(4140) = \{D_s^*D_s^{*\dagger}\}$ are of order of 1 keV for $\Lambda_{H_i} = 2$ GeV.

For completeness, we compare the two-photon decays of the heavy systems to the one of the light scalars f_0/a_0. The stability of the coupling in case of a $K\bar{K}$ bound state already implies that the radiative decay width $\Gamma(f(980) \to \gamma\gamma)$ is not sensitive to the cutoff or finite-size effects provided Λ_{H_i} is above 0.5 GeV. Here we completely agree with the conclusions of Ref. [15]). In particular, in order to reproduce the current data on strong and radiative decays of $f(980)$ we fixed the cutoff parameter $\Lambda_{H_K} \equiv \Lambda_f = 1$ GeV [6] with

$$\Gamma(f(980) \to \gamma\gamma) = 0.25 \text{ keV} \tag{18}$$

a value which is very close to the result of the local approximation:

$$\Gamma(f(980) \to \gamma\gamma) = 0.29 \text{ keV}. \tag{19}$$

Similar results are obtained by the molecular approach in [15] and [19]. However, for smaller values of $\Lambda_f < 0.6$ GeV the radiative decay width decreases which means that such small values of the cutoff parameter Λ_f are unlikely according to present data.

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